



Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

II. *A Rule for trisectioning Angles geometrically*, by JAMES WINTHROP, Esq.

THE trisection of angles, by a geometrical process, has been so long sought for by the curious, that I cannot help feeling some degree of reluctance at offering any thing on that subject. As I am wholly unacquainted with what is already done on this question, it is not impossible, that the following solution of it may have been already published. Should this happen to be the case, I hope that the candour of this learned society will excuse it, as arising from want of opportunity to read in these branches of science, and the consequent want of opportunity to get information in them.

Let A B or B g (Plate 1. Fig. 5.) represent the arcs to be divided, and A f or g f, one third of their respective arcs ; then e f will be in each of them, the sine of the third, and B f the chord of the two thirds. Now, by the properties of a circle, the sine e f must always be equal to half the chord f B. The difficulty consists in ascertaining the point f, whence the sine and the chord must be drawn, bearing that relation to each other. The method to be proposed at present, is by a regular curve which shall trisect all possible arcs, having their centres in the line A C, and terminating one way in that line, and the other way in the point B.

It is evident, that any arc, less than a quadrant, may be drawn from B to the radius A d of the primitive circle (Fig. 5.) as by the construction of that figure, B g drawn round C as a centre, is equal to half a right angle. It is likewise clear, that

that the nearer the arc is drawn to the line Bd , the less portion of a circle it will be, till at last it degenerates into a right line. Let dh be made equal to one third of Bd ; and the sine ef will always exceed dh , till the arc shall be reduced to a right line; when they will be exactly equal. A curve therefore, having its vertex in h , and so constructed as to pass through f , or, which is the same thing, having every point of its circumference twice as far from B as from the line Ad , will trisect all possible arcs having their centres in the line AC . For, by being continued from h to f , it will trisect the smaller portion of a circle, or a portion less than a semicircle, which has its centre in the line AC ; and if it be continued infinitely from V through C (Fig. 6.) it will trisect the greater portion.

Having ascertained the property of the curve which applies to our present purpose, it remains to propose methods for projecting it. The methods we shall take are the same that are used in drawing a parabola. We shall begin with finding points.

Draw the line AF (Fig. 6) of a convenient length. Make AV equal to half of FV ; draw AB equal to twice AF , and at right angles with AF ; and complete the parallelogram $ABCF$. Then will F be the focus, V the vertex, AB the directrix, and FC the semiparameter of the curve. To find a suitable number of points in the circumference of the curve, extend the compasses from A to some point as d , in the line AF , between F and V . Make BG
equal

equal to $A d$, and join $d G$. Double the distance $A d$, and set off that new distance from F to P , where the compasses mark the line $d G$. The point P will be in the periphery of the curve. For let $P I$ be drawn perpendicular to the directrix $A B$; then, because $d G$ is parallel to $A B$, and the angle at A is a right angle, $P I$ is equal to $A d$; and because $F P$ is double to $A d$, it is also double to $P I$; which was required.

The trouble of drawing this curve is very little more than that of drawing the parabola. The additional labour consists only in doubling the distance $A d$, in order to find the *Radius Vector* $F P$. In the parabola, $A V$ is always equal to the focal distance $F V$; and the radius vector is always equal to the focal distance, *plus* the absciss $V d$. In the trisection curve, the radius vector is always equal to twice $A d$; or, which is the same thing, as $A V$ is equal to half the focal distance, the radius vector $F P$ is equal to the whole focal distance, *plus* twice the absciss $V d$. Hence it appears, that $F C$, the semiparameter of the trisection curve, is to the semiparameter of a parabola of the same focal distance, as three to two.

If an arc be described less than a quadrant as $B f g$ (Fig. 5.) it will be so divided, that $f g$ will be one third of it. But $f g$ will be one sixth of any arc $B g k$, less than a semicircle. Any arc therefore, less than a semicircle, will be trisected by that part of the curve which falls within the primitive circle. The greater portion of a circle may be trisected, either

er by substracting one third of the smaller portion from a third of the whole, or by continuing the arc through the point F, till it meets the curve without the primitive circle. For F P being one third of a semicircle, the arc intercepted between F and any point of the curve without the point P, will be one third of a portion larger than a semicircle.

In fig. 7, is represented the manner of describing the trifecting curve by a steady motion. A B is the directrix ; F C, the arc to be divided ; D E G, a square, one side of which is to be moved along the directrix ; F E, part of a thread, one end of which is tied to a pin at the focus F, and the length of it such, that when the side of the ruler, along which the pencil is to move, coincides with F V, the axis of the curve, and the pencil placed in V, the thread is to pass from F round the pencil, then round a pin near the top of the ruler, and returning, the end is to be tied to the pencil. Then, as one side of the square moves along the directrix, the *radius vector* F E is continually lengthening ; and of consequence, the pencil is pressed towards the top of the ruler. Upon the principle of the pulley, the line F E must increase twice as fast as the pencil ascends towards the upper extremity of the ruler ; which is precisely the proportion required. This construction differs from that of the parabola, only in tying one end of the thread to the pencil, instead of fastening it to the ruler ; and this difference is necessary, because in the present case, the additions to the radius vector are double to those in the parabola.

C

III. Rules